Nova Scotia Examinations
Advanced Mathematics 12
Web Sample 2

Student Booklet

NOVA SCOTIA

Education
Evaluation Services

General Instructions - WEB SAMPLE*

This examination is composed of two sections with the following suggested time allotment:

Selected-Response (Multiple-Choice) Questions   Value 25 pts   (approx. 40 min)*
Constructed-Response Questions   Value 75 pts   (approx. 120 min)

*note: there are 35 constructed response questions on the Math NSEs as of January 2008

Total time: 3 hours (revision time included)

Use these suggested times to guide you in the completion of the examination; however, you might not find it necessary to spend the suggested time on each section. Plan your time to enable you to complete the examination.

You are not permitted to use your own graphing calculator unless your teacher has cleared the memory immediately prior to this examination. The only graphing calculators permitted are TI-82, TI-83, TI-83 Plus, TI-84, or TI-84 Plus.

If the question indicates that you are not to use a graphing calculator, you are still permitted to use a calculator to perform arithmetic operations.

Calculators are not to be shared.

Graph paper, scrap paper, and formula sheets are provided at the end of this booklet. These pages can be removed from the booklet for your use during the examination.

Note: Diagrams are not necessarily drawn to scale.

Selected-Response Questions - WEB SAMPLE
(Total Value: 25 points)

In this part of the examination, there are 25 selected-response questions*, each with a value of 1 point. Read each question carefully, and decide which of the responses best answers the question being asked.

You are provided a separate student answer sheet. In the selected-response section of the student answer sheet, fill in the bubble that corresponds to your choice as shown in the example below. Use an HB pencil only.

Example

1. What are the roots of $x^2 + 3x - 4 = 0$?

   A. 4 and 1   B. −4 and 3
   C. −4 and 1   D. 4 and 3

   (On student answer sheet)

   1.   Ⓐ Ⓑ ⬤ Ⓓ

If you wish to change an answer, please ensure that you erase your first answer completely on the student answer sheet. Calculations or rough work on the selected-response pages of the examination booklet will not be scored.

*Note: As of January 2008 there are 35 selected response questions on the NSE Math exams.

1. The function $t_n = 2n^2 + 3n - 1$ is used to generate a quadratic sequence. What is the value of D_2 ?

   A. 1   B. 2
   C. $-\frac{3}{4}$ D. 4

2. The axis of symmetry for the parabola defined by $y = x^2 - 6x + 12$ is

   A. $x = -4$ B. $x = -6$
   C. $x = 3$ D. $x = 12$

3. A cannon ball is shot into the air and its height, in metres, is represented by the function $h = 1.5 + 13.1t - 4.9t^2$ where 't' is time in seconds. Which of the following best approximates how long it will take for the ball to hit the ground?

   A. 1.34 seconds   B. 1.50 seconds
   C. 2.78 seconds   D. 10.25 seconds

4. If the roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-1 \pm i\sqrt{19}}{10}$, then what is true of the graph of the function $y = ax^2 + bx + c$?

   A. has no x-intercept   B. has no y-intercept
   C. is not parabolic   D. has 2 distinct x-intercepts

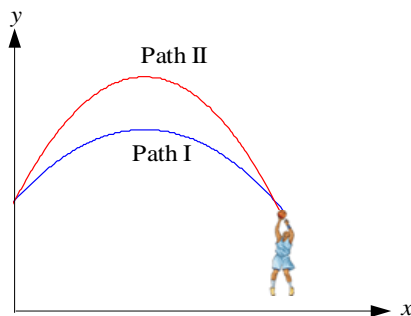5. The value of c that will make the expression $9x^2 - 12x + c$ a perfect square is

   A. 36   B. 4
   C. 81   D. 16

6. Given $4x^2 + 4x + k = 0$, for what values of k will there be no real roots?

   A. $k < 1$ B. $k > 1$
   C. $k < 0$ D. $k > -1$

7. A basketball player is trying to increase her shot accuracy. She stays in the same position on the court and increases the arc of the flight path of the ball.

Her coach graphs the quadratic function $\frac{1}{a}(y-k)=(x-h)^2$ to model parabolic path I of the basketball. The coach then changes certain values in the given equation to graph path II. Which value(s) did the coach NOT change ?

A. a   B. h
C. k   D. h and a

---

8. Which one of the following tables of values was generated by an exponential function?

A.

x	−1	2	3	5
y	$\frac{1}{2}$	1	2	4

B.

x	−1	1	3	5
y	1	3	9	27

C.

x	−1	0	1	2
y	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$

D.

x	1	2	3	4
y	1	4	9	16

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9. The expression $3^k + 3^k + 3^k$ equals

A. 3^{3k} B. 9^k
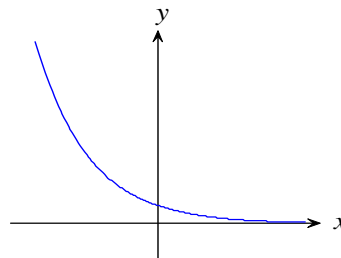
C. 3^{k+1} D. 9^{3k}

10. Which graph represents $y = a \cdot b^x$, where $a > 0$ and $0 < b < 1$?
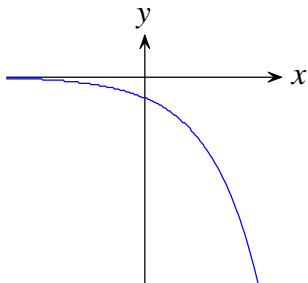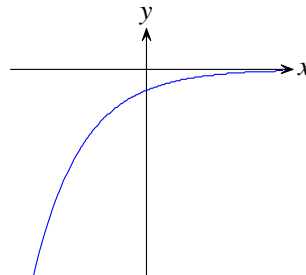
A.
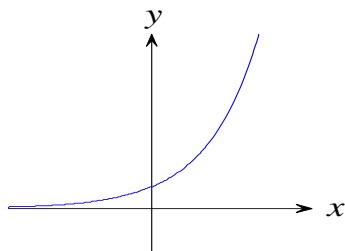
B.

C.

D.

11. After certain transformations, the image of $y = 3^x$ is $y = 2(3)^{-2x}$. The negative sign in the exponent is a result of a

A. reflection in the *x*-axis
B. reflection in the *y*-axis
C. horizontal translation to the left
D. vertical stretch downwards

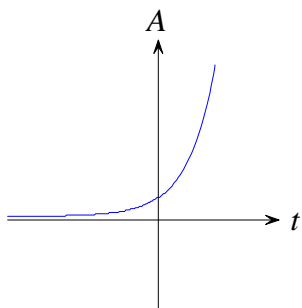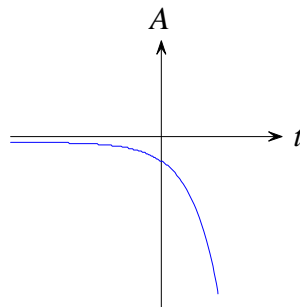12. The expression $\dfrac{\log_2 16}{\log_2\left(\frac{1}{2}\right)}$ is equal to

A. –4
B. 3
C. 5
D. 32

13. Which of the following graphs could represent the remaining amount, A, of a radioactive substance as it decays over time t in years?
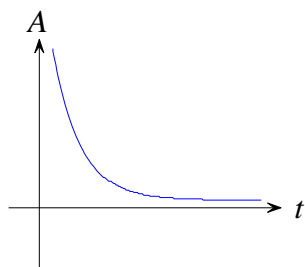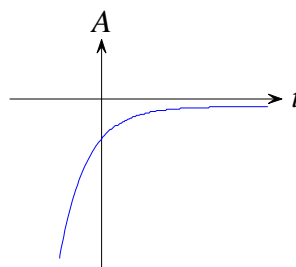
A.

B.

C.

D.

14. Which of the following is FALSE?

A. $\dfrac{\log_a M}{\log_a N} = \log_a M - \log_a N$

B. $\dfrac{\log_a M}{c} = \log_a M^{\frac{1}{c}}$

C. $\log_M N = \dfrac{\log N}{\log M}$

D. $\log_N (MN)^x = x \log_N M + x$

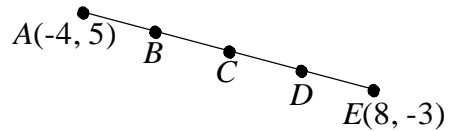15. What is the value of x in $\log_8 x = \dfrac{5}{3}$?

A. 3.48

B. $\dfrac{40}{3}$

C. 32

D. 16

16. The segment below is to be cut into 4 equal pieces.

What are the coordinates of *B*?

A. $(2,1)$

B. $(-1,2)$

C. $(1,\frac{1}{2})$

D. $(-1,3)$

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17. If two chords of a circle intersect, then their perpendicular bisectors

A. don't intersect

B. are parallel

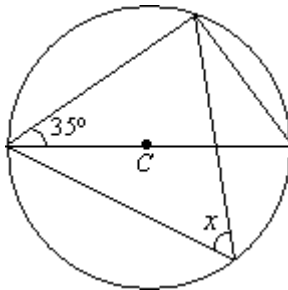C. bisect each other

D. intersect at the centre of the circle

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18. Which is the general form of $(x-3)^2+(y+1)^2=1$?

A. $x^2+y^2-6x+2y+11=0$

B. $x^2+y^2-6x+2y+9=0$

C. $x^2+y^2+6x-2y+9=0$

D. $x^2+y^2+9=0$

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19. The equation of an ellipse is $x^2+4y^2=36$. Its domain is

A. $-3\le x\le 3$

B. $-12\le x\le 12$

C. $-6\le x\le 6$

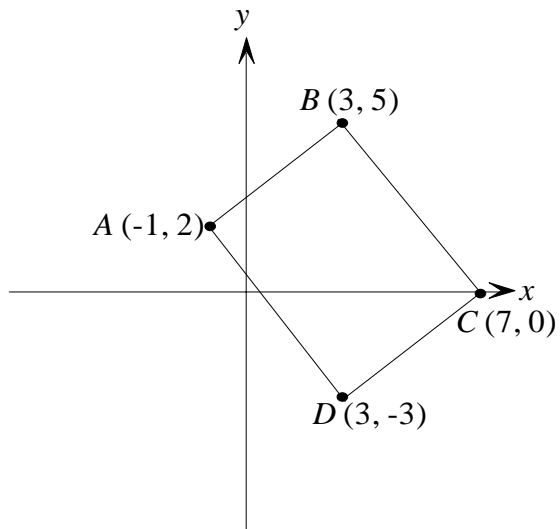D. $-18\le x\le 18$

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20. C is the centre of the following circle.

The value of *x* is

A. $90°$

B. $45°$
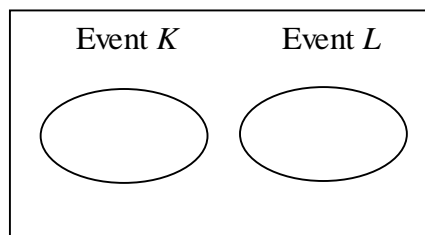
C. $55°$

D. $35°$

21. Given the following figure:

Which of the following statements is **false**?

A. $AB = DC$

B. The midpoint of $\overline{AD}$ is $(1, -\frac{1}{2})$.

C. The diagonals $\overline{AC}$ and $\overline{BD}$ are of equal lengths.

D. The diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other.

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22. The value of $\dfrac{500!}{499!}$ is

A. 1

B. 1.002

C. 500

D. undefined

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23. Consider the following Venn diagram.

Which of the following is correct?

A. P(K or L) = P(K) + P(L)

B. P(K and L) = P(K) + P(L)

C. P(K or L) = P(K) × P(L)

D. P(K and L) = P(K) × P(L)

24. Your math teacher gives your class a list of eight questions to study. Five of the eight questions will be randomly selected for the next test. If you study only the first five questions from the list, the probability that all of those five questions will be on the test is

A. $\dfrac{1}{_8C_5}$

B. $\dfrac{1}{_8P_5}$

C. $\dfrac{5}{8}$

D. $\dfrac{1}{8}$

25. There are 5 red marbles and 7 blue marbles in a bag. Two marbles are chosen randomly without replacement. The probability that a red and then a blue marble will be chosen is

A. $\dfrac{5}{12} \times \dfrac{7}{11}$

B. $\dfrac{1}{_5C_1 \times _7C_1}$

C. $\dfrac{5}{12} + \dfrac{7}{11}$

D. $\dfrac{1}{_{12}C_2}$

Constructed-Response Questions
(Total Value: 75 points)

Read each question carefully, and be sure to write your response in the box and space provided. If the answer box indicates that you are to show your work, then points will be awarded for your correct work and your correct final answer. The method used to solve a problem must clearly be shown even when using a graphing calculator. If the answer box requires that just a final answer be provided, then points will be awarded for the correct answer only.

When working with decimal values, you may round off to the hundredths place in your final answer only. If any decimal values are rounded prior to the final step of the solution, at least 4 decimal places must be kept.

With the exception of the probability unit, all answers must be given in simplified form.

26. The severity of an automobile crash increases significantly as the speed increases. The table shows the relationship between the speed and a crash severity index.

Speed (km/h)	10	20	30	40	50
Crash severity index	1.20	4.40	9.60	16.80	26.00

(a) Jimmy claims that a quadratic function would best model this situation. Is Jimmy's claim correct? Explain. (2 points)

(b) What speed would have a crash severity index of 50.40? (3 points)

Show your work above and write your conclusion or final answer in the box below.

27. Solve for x in each of the following equations using a different algebraic method for each. Express your answers in simplified form.

(a) $x^2 = x + 6$ (3 points)

Show your work above and write your conclusion or final answer in the box below.

(b) $2x^2 + 2x + 5 = 0$ (3 points)

Show your work above and write your conclusion or final answer in the box below.

28. A football is kicked into the air. The equation $h = -4.9t^2 + 9.8t + 1$ expresses the relationship between height, h, in metres and time, t, in seconds.

(a) Determine the maximum height reached by the football. (3 points)

Show your work above and write your conclusion or final answer in the box below.

(b) For how long was the ball at a height of at least 5 m above the ground? (3 points)

Show your work above and write your conclusion or final answer in the box below.

29. A golf ball is hit from ground level in a flat field and reaches a maximum height of 25 m. The ball first hits the ground 100 m away while following a parabolic path.

(a) Draw a diagram and include all important information needed to model this problem. Remember to label your axes. (2 points)

(b) How high is the golf ball above the ground when it is at a horizontal distance of 20 m from where it was hit? (4 points)

Show your work above and write your conclusion or final answer in the box below.

30. Prove that there are no real values of m for which the quadratic equation $x^2 - 5 = 2m(x-2)$ has equal roots. (4 points)

31. Find the zero(s) of the function $f(x) = 2^{3x+5} - 7$. (3 points)

Show your work above and write your conclusion or final answer in the box below.

32. Algebraically solve for x.

(a) $(\sqrt{2})^{3x} = 16^{x-2}$ (2 points)

> Show your work above and write your conclusion or final answer in the box below.

(b) $\log_x 18 - \log_x 2 + \log_x 3 = \log_5 125$ (2 points)

> Show your work above and write your conclusion or final answer in the box below.

(c) $\log_4(x+3) - \log_4 x = 2$ (2 points)

> Show your work above and write your conclusion or final answer in the box below.

33. Given $\frac{1}{2}(y-4)=2^{x-3}$.

   (a) Sketch the graph of the given function. Clearly state the coordinates of the y-intercept and two other points on the graph. Sketch the asymptote and provide the equation of the asymptote. (3 points)
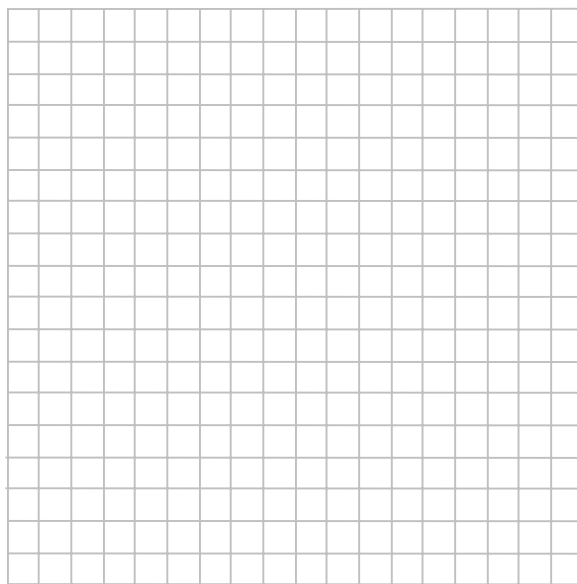
   (b) Indicate whether the function above represents a growth curve or a decay curve. Explain how you know. (1 point)

   (c) State the domain and range of the function. (1 point)

   Range: | Final Answer |

   Domain: | Final Answer |

34. The population of country A is 40 million on January 1, 1990 and increases by 6% each year thereafter. The population of country B is 60 million on January 1, 1990 and increases by 4% each year thereafter. When will the populations be the same? (5 points)

Show your work above and write your conclusion or final answer in the box below.

35. (a) Evaluate: (1 point)

    (i) $\log_2 8 =$ _____

    (ii) $- \log_{\frac{1}{2}} 8 =$ _____

  (b) Evaluate: (1 point)

    (i) $\log_3 9 =$ _____

    (ii) $- \log_{\frac{1}{3}} 9 =$ _____

  (c) Based on the answers obtained in parts (a) and (b), write an expression equivalent to $- \log_{\frac{1}{b}} N$. (1 point)

Final Answer	

36. Describe a situation that could be modelled by the function $P = 1000(1.03)^{2t}$. (3 points)

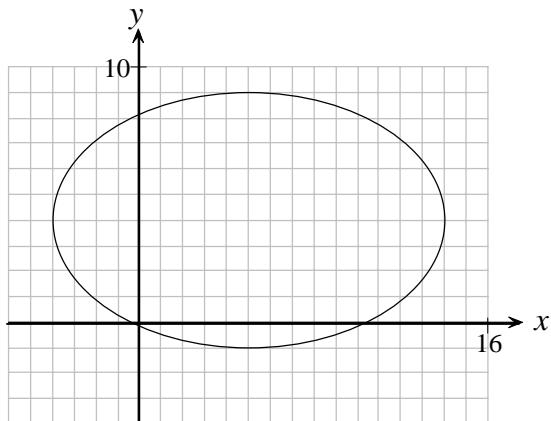37. Complete each of the following statements.   (2 points)

   (a) Chords in the same circle are of equal length iff_____

   _____

   (b) A line is a perpendicular bisector of a chord iff_____

   _____

38. Write the equation of the following graph in transformational form.   (3 points)

Show your work above and write your conclusion or final answer in the box below.

39. The endpoints of a diameter of a circle are A(-1, 3) and B(13, 5). Determine if the point C(11, 9) is on the circumference of the circle. Clearly show your reasoning. (4 points)

Show your work above and write your conclusion or final answer in the box below.

40. Given: $\overline{PO} \cong \overline{SO}$ (5 points)

$\overline{OQ} \cong \overline{OR}$

Prove: $\triangle PQR \cong \triangle SRQ$
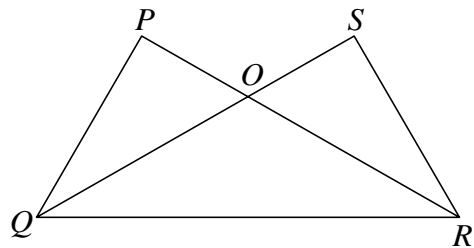
41. There are 15 jellybeans randomly distributed in a jar; 5 are yellow and 10 are orange. You reach into the jar and, without looking, remove 2 jellybeans. What is the probability that you will remove 2 yellow jellybeans? (3 points)

Show your work above and write your conclusion or final answer in the box below.

42. In a group of 15 people, 4 are left-handed and 11 are right-handed. Seven people are selected at random from this group.

(a) What is the probability that all 4 left-handed people will be selected?   (2 points)

Show your work above and write your conclusion or final answer in the box below.

(b) If Sarah and Mike, two of the left handers, have already been chosen, what is the probability that all the other members selected will be right-handed?   (2 points)

Show your work above and write your conclusion or final answer in the box below.

43. Create a real-life problem that demonstrates P(A or B) = P(A) + P(B) $-$ P(A and B) when the events A and B are NOT mutually exclusive.
   (You don't have to solve the problem.) (2 points)

You have reached the end of the ADVANCED WEB SAMPLE 2 Examination.
Please check your work to ensure you have completed all questions.

Advanced Mathematics 12

Formula Sheet – Advanced Mathematics 12

Quadratic Unit

General form: $y = ax^2 + bx + c$

Standard form: $y = a(x-h)^2 + k$

Transformational form: $\dfrac{1}{a}(y-k) = (x-h)^2$

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Exponential Growth Unit

$y = ab^x$

$A(y-C) = b^{B(x-D)}$

$\log_a(xy) = \log_a x + \log_a y$

$\log_a(x \div y) = \log_a x - \log_a y$ or $\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^b = b(\log_a x)$

Circle Geometry Unit

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The coordinates of M are: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

General form: $Ax^2 + Ay^2 + Dx + Ey + F = 0$

$$Ax^2 + By^2 + Dx + Ey + F = 0$$

Standard form: $(x - h)^2 + (y - k)^2 = r^2$

Transformational form: $\left[\dfrac{1}{r}(x - h) \right]^2 + \left[\dfrac{1}{r}(y - k) \right]^2 = 1$

$$\left[\dfrac{1}{a}(x - h) \right]^2 + \left[\dfrac{1}{b}(y - k) \right]^2 = 1$$

$$m = \dfrac{\Delta y}{\Delta x}$$

Probability Unit

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)}$$

$$_nP_r = \dfrac{n!}{(n-r)!} \qquad\qquad _nC_r = \dfrac{n!}{r!(n-r)!}$$

Nova Scotia Examinations

Advanced Mathematics 12

Web Sample 2

Marking Guide

NOVA SCOTIA

Education
Evaluation Services

Teacher
Name: _____

Selected Response Answers

1.	D	14.	A
2.	C	15.	C
3.	C	16.	D
4.	A	17.	D
5.	B	18.	B
6.	B	19.	C
7.	B	20.	C
8.	B	21.	C
9.	C	22.	C
10.	B	23.	A
11.	B	24.	A
12.	A	25.	A
13.	C		

Question 26 (a) (2 points)

Yes. D_2 is constant. ($D_2 = 2$, see table below)

[1 pt]
[1 pt]

OR

Yes. $R^2 = 1$ using quadratic regression.

Question 26 (b) (3 points)

$y = ax^2 + bx + c$

Quad Reg on TI-83

$200a = 2$
$a = 0.01$ [0.5 pt]

$y = 0.01x^2 + 0.02x$ [1 pt]

$c = 0$ [0.5 pt]

Using Table

$(10,\ 1.20) \Rightarrow \quad 1.20 = 0.01(10)^2 + 10b + 0$
$1.20 = 1 + 10b$
$0.20 = 10b$
$0.02 = b$

[0.5 pt]

69.99	50.39
70.00	50.4
70.01	50.41

[1 pt]

$\therefore x = 70$ [1 pt]

$0.01x^2 + 0.02x = 50.4$ [0.5 pt]
$0.01x^2 + 0.02x - 50.4 = 0$

Solve for x using any method to
get $x = 70$. [1 pt]

OR

10	20	30	40	50	60	70
1.2	4.4	9.6	16.8	26	37.2	50.4

[1 pt]

3.2	5.2	7.2	9.2	11.2	13.2

[1 pt]

$\therefore x = 70$ [1 pt]

2

$$x^2 = x + 6$$

$$x^2 - x - 6 = 0 \quad \boxed{\text{0.5 pt}}$$

$$\boxed{\text{1 pt}} \quad (x-3)(x+2) = 0 \quad \boxed{\text{1 pt}}$$

$x - 3 = 0$	$x + 2 = 0$
$x = 3$	$x = -2$

$\boxed{\text{0.5 pt}}$ both answers must be correct

$$x^2 = x + 6$$

$$x^2 - x - 6 = 0 \quad \boxed{\text{0.5 pt}}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - (4)(1)(-6)}}{(2)(1)} \quad \boxed{\text{1 pt}}$$

$$= \frac{1 \pm \sqrt{25}}{2} \quad \boxed{\text{0.5 pt}}$$

$x = \dfrac{1+5}{2}$	$x = \dfrac{1-5}{2}$
$= 3$	$= -2$
$\boxed{\text{0.5 pt}}$	$\boxed{\text{0.5 pt}}$

◇ OR ◇

$$x^2 = x + 6$$

$$x^2 - x = 6 \quad \boxed{\text{0.5 pt}}$$

$$x^2 - x + \frac{1}{4} = 6 + \frac{1}{4} \quad \boxed{\text{0.5 pt}}$$

$$\left(x - \frac{1}{2}\right)^2 = \frac{25}{4} \quad \boxed{\text{0.5 pt}}$$

$$x - \frac{1}{2} = \pm\sqrt{\frac{25}{4}}$$

$$x = \frac{1}{2} \pm \frac{5}{2} \quad \boxed{\text{0.5 pt}}$$

$x = \dfrac{1}{2} + \dfrac{5}{2}$	$x = \dfrac{1}{2} - \dfrac{5}{2}$
$= 3$	$= -2$
$\boxed{\text{0.5 pt}}$	$\boxed{\text{0.5 pt}}$

Question 27 (b)   (3 points)

$2x^2 + 2x + 5 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - (4)(2)(5)}}{(2)(2)}$$ `1 pt`

$$= \frac{-2 \pm \sqrt{-36}}{4}$$ `0.5 pt`

$$= \frac{-2 \pm 6i}{4}$$ `1 pt`

$$= \frac{-1 \pm 3i}{2}$$ `0.5 pt`

OR

$2x^2 + 2x + 5 = 0$

$$x^2 + x + \frac{5}{2} = 0$$ `0.5 pt`

$$x^2 + x = -\frac{5}{2}$$

$$x^2 + x + \frac{1}{4} = -\frac{5}{2} + \frac{1}{4}$$ `0.5 pt`

$$\left(x + \frac{1}{2}\right)^2 = -\frac{9}{4}$$ `1 pt`

$$x + \frac{1}{2} = \pm\sqrt{-\frac{9}{4}}$$ `0.5 pt`

$$x = -\frac{1}{2} \pm \frac{3}{2}i \text{ or } \frac{-1 \pm 3i}{2}$$ `0.5 pt`

4

Question 28 (a) (3 points)

$$h = -4.9t^2 + 9.8t + 1$$
$$h - 1 = -4.9t^2 + 9.8t \quad \boxed{0.5 \text{ pt}}$$
$$h - 1 = -4.9(t^2 - 2t) \quad \boxed{0.5 \text{ pt}}$$
$$h - 1 - 4.9 = -4.9(t^2 - 2t + 1) \quad \boxed{0.5 \text{ pt}}$$
$$-\frac{1}{4.9}(h - 5.9) = (t - 1)^2 \quad \boxed{0.5 \text{ pt}}$$

The maximum height is 5.9 m. $\boxed{1 \text{ pt}}$

$$t = -\frac{b}{2a}$$
$$= \frac{-9.8}{2(-4.9)}$$
$$= 1 \quad \boxed{1 \text{ pt}}$$

$$h(1) = -4.9(1)^2 + 9.8(1) + 1 \quad \boxed{1 \text{ pt}}$$
$$= 5.9$$

$\boxed{1 \text{ pt}}$

The maximum height is 5.9 m.

OR

$$y = -4.9x^2 + 9.8x + 1$$

(1, 5.9)

$\boxed{2 \text{ pts}}$

The maximum height is 5.9 m. $\boxed{1 \text{ pt}}$

5

$-4.9t^2 + 9.8t + 1 = 5$

$-4.9t^2 + 9.8t - 4 = 0$ — 0.5 pt

$t = \dfrac{-9.8 \pm \sqrt{(9.8)^2 - (4)(-4.9)(-4)}}{(2)(-4.9)}$ — 1 pt

$= 0.57$ and 1.43 — 0.5 pt

Time above 5 m $= 1.43 - 0.57$

$= 0.86$ — 1 pt

$y = -4.9t^2 + 9.8t - 4$ — 0.5 pt

(0.57, 0)   (1.43, 0)
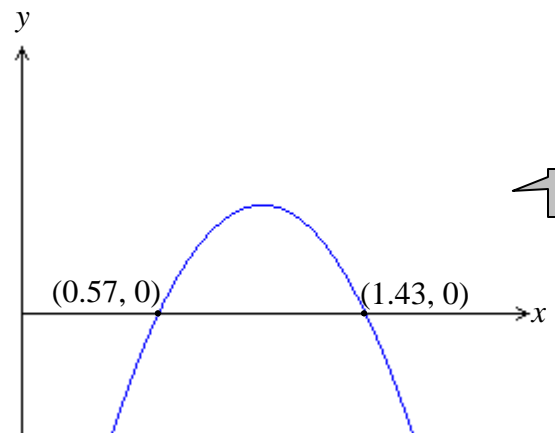
— 1.5 pts

Time above 5 m $= 1.43 - 0.57$

$= 0.86$ — 1 pt

OR

$\left.\begin{array}{l} y_1 = -4.9x^2 + 9.8x + 1 \\ y_2 = 5 \end{array}\right\}$ — 0.5 pt

(0.57, 5)   (1.43, 5)

— 1.5 pts

Time above 5 m $= 1.43 - 0.57$

$= 0.86$ — 1 pt

6

Question 29 (a)   (2 points)

(50, 25)   0.5 pt

0.5 pt

(0, 0)   0.5 pt

(100, 0)

t 0.5 pt
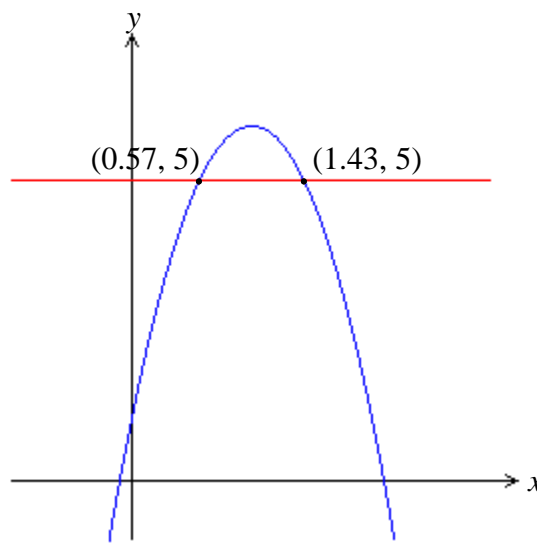
For axis labels
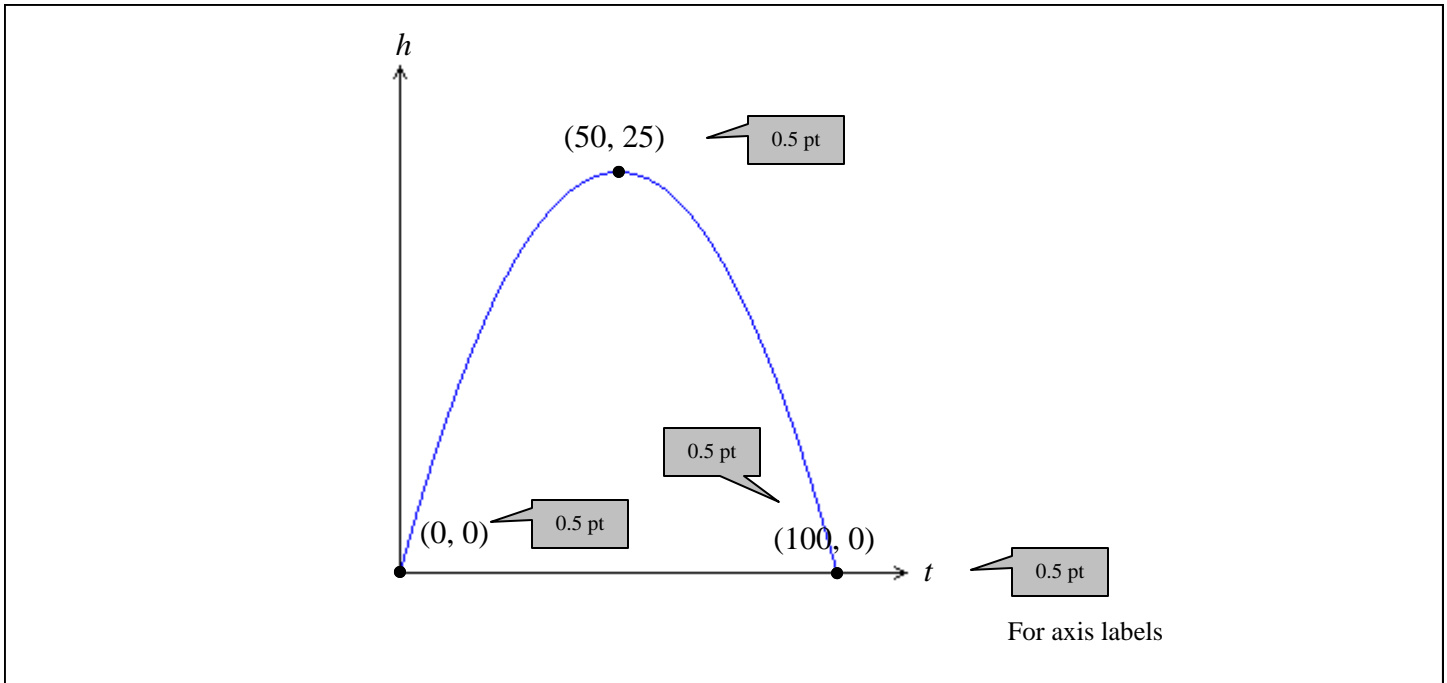
Question 29 (b)   (4 points)

Using regression on TI-83   1 pt

$y = -0.01x^2 + x$ 1 pt

$y = -0.01(20)^2 + 20$ 1 pt

$= 16$ 1 pt

The golf ball is 16 m high.

Using regression on TI-83   1 pt

$y = -0.01x^2 + x$ 1 pt

x	y
19	15.39
20	16 1 pt
21	16.59

The golf ball is 16 m high.   1 pt

OR

$\dfrac{1}{a}(y - 25) = (x - 50)^2$ 1 pt

0.5 pt

$(0, 0) \Rightarrow \dfrac{-25}{a} = (-50)^2$

$a = -\dfrac{25}{2500}$

$a = -0.01$ 0.5 pt

$-\dfrac{1}{0.01}(y - 25) = (x - 50)^2$ 0.5 pt

$-\dfrac{1}{0.01}(y - 25) = (20 - 50)^2$ 0.5 pt

$-100(y - 25) = 900$

$y - 25 = -9$

$y = 16$ 1 pt

The golf ball is 16 m high.

7

Question 30   (5 points)

$$x^2 - 5 = 2m(x - 2)$$
$$x^2 - 2m(x - 2) - 5 = 0$$
$$x^2 - 2mx + 4m - 5 = 0 \quad \boxed{0.5 \text{ pt}}$$

If equal roots

$$b^2 - 4ac = 0 \quad \boxed{0.5 \text{ pt}}$$
$$(-2m)^2 - 4(1)(4m - 5) = 0 \quad \boxed{0.5 \text{ pt}}$$
$$4m^2 - 16m + 20 = 0 \quad \boxed{0.5 \text{ pt}}$$
$$m^2 - 4m + 5 = 0$$

$$m = \frac{4 \pm \sqrt{16 - 4(1)(5)}}{2}$$
$$= \frac{4 \pm \sqrt{-4}}{2}$$
$$= \frac{4 \pm 2i}{2}$$
$$= 2 \pm i \quad \boxed{1 \text{ pt}}$$

OR

$$\text{Discriminant} = b^2 - 4ac$$
$$= (-4)^2 - (4)(1)(5)$$
$$= 16 - 20$$
$$= -4$$

The roots are not real. $\quad \boxed{1 \text{ pt}}$

There are no real numbers for which $b^2 - 4ac = 0$; therefore there are no values of m such that the equation has equal roots. $\quad \boxed{1 \text{ pt}}$

8

$2^{3x+5} - 7 = 0$

$2^{3x+5} = 7$ — 0.5 pt

$\log 2^{3x+5} = \log 7$ — 0.5 pt

$(3x+5)\log 2 = \log 7$ — 0.5 pt

$3x\log 2 + 5\log 2 = \log 7$

$x = \dfrac{\log 7 - 5\log 2}{3\log 2}$ — 0.5 pt

$x = -0.73$ — 1 pt

$2^{3x+5} - 7 = 0$

$2^{3x+5} = 7$ — 0.5 pt

$3x + 5 = \log_2 7$ — 0.5 pt

$x = \dfrac{\log_2 7 - 5}{3}$ — 1 pt

$x = -0.73$ — 1 pt

OR

$y = 2^{3x+5} - 7$

2 pts

$(-0.73, 0)$

$\therefore x = -0.73$ — 1 pt

$\left. \begin{array}{l} y_1 = 2^{3x+5} \\ y_2 = 7 \end{array} \right\}$ — 1 pt

$(-0.73, 7)$

1 pt

$\therefore x = -0.73$ — 1 pt

9
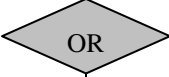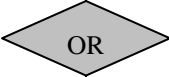
Question 32 (a) (2 points)

Left solution:

$$\left(\sqrt{2}\right)^{3x} = 16^{x-2}$$

$$\left(2^{\frac{1}{2}}\right)^{3x} = \left(2^4\right)^{x-2} \quad \text{0.5 pt}$$

$$2^{\frac{3x}{2}} = 2^{4x-8} \quad \text{0.5 pt}$$

$$\therefore \frac{3x}{2} = 4x - 8 \quad \text{0.5 pt}$$

$$3x = 8x - 16$$

$$-5x = -16$$

$$x = \frac{16}{5} \quad \text{0.5 pt}$$

OR

Right solution:

$$\left(\sqrt{2}\right)^{3x} = 16^{x+2}$$

$$\log\left(\sqrt{2}\right)^{3x} = \log 16^{x+2} \quad \text{0.5 pt}$$

$$3x\log\sqrt{2} = (x-2)\log 16 \quad \text{0.5 pt}$$

$$3x\log\sqrt{2} - x\log 16 = -2\log 16$$

$$x\left(3\log\sqrt{2} - \log 16\right) = -2\log 16$$

$$x = \frac{-2\log 16}{3\log\sqrt{2} - \log 16} \quad \text{0.5 pt}$$

$$x = 3.2 \quad \text{0.5 pt}$$

Question 32 (b) (2 points)

Left solution:

$$\log_x 18 - \log_x 2 + \log_x 3 = \log_5 125$$

$$\log_x\left(\frac{(18)(3)}{2}\right) = 3 \quad \text{1 pt}$$

$$\log_x 27 = 3$$

$$x^3 = 27 \quad \text{0.5 pt}$$

$$x = 3 \quad \text{0.5 pt}$$

OR

Right solution:

$$\log_x - \log_x 2 + \log_x 3 = \log_5 125$$

$$\frac{\log 18}{\log x} - \frac{\log 2}{\log x} + \frac{\log 3}{\log x} = 3 \quad \text{0.5 pt}$$

$$\log 18 - \log 2 + \log 3 = 3\log x \quad \text{0.5 pt}$$

$$1.4313 = 3\log x$$

$$\frac{1.4313}{3} = \log x$$

$$10^{0.4771} = x \quad \text{0.5 pt}$$

$$3 = x \quad \text{0.5 pt}$$

$$\log_4 (x+3) - \log_4 x = 2$$

$$\log_4 \left(\frac{x+3}{x} \right) = 2 \quad \boxed{0.5 \text{ pt}}$$

$$\frac{x+3}{x} = 4^2 \quad \boxed{0.5 \text{ pt}}$$

$$x + 3 = 16x$$

$$3 = 15x \quad \boxed{0.5 \text{ pt}}$$

$$\frac{1}{5} = x \quad \boxed{0.5 \text{ pt}}$$

$$\log_4 (x+3) - \log_4 x = 2$$

$$\log_4 (x+3) - \log_4 x = \log_4 16 \quad \boxed{0.5 \text{ pt}}$$

$$\log_4 \left(\frac{x+3}{x} \right) = \log_4 16 \quad \boxed{0.5 \text{ pt}}$$

$$\therefore \ \frac{x+3}{x} = 16 \quad \boxed{0.5 \text{ pt}}$$

$$x + 3 = 16x$$

$$3 = 15x$$

$$\frac{1}{5} = x \quad \boxed{0.5 \text{ pt}}$$

OR

$$\log_4 (x+3) - \log_4 x = 2$$

$$\frac{\log (x+3)}{\log 4} - \frac{\log x}{\log 4} = 2 \quad \boxed{0.5 \text{ pt}}$$

$$\log (x+3) - \log x = 2 \log 4 \quad \boxed{0.5 \text{ pt}}$$

$$\log \left(\frac{x+3}{x} \right) = 2 \log 4$$

$$\frac{x+3}{x} = 10^{2 \log 4} \quad \boxed{0.5 \text{ pt}}$$

$$x + 3 = 10^{2 \log 4} x$$

$$3 = 15x$$

$$\frac{1}{5} = x \quad \boxed{0.5 \text{ pt}}$$

Question 33 (a)   (3 points)
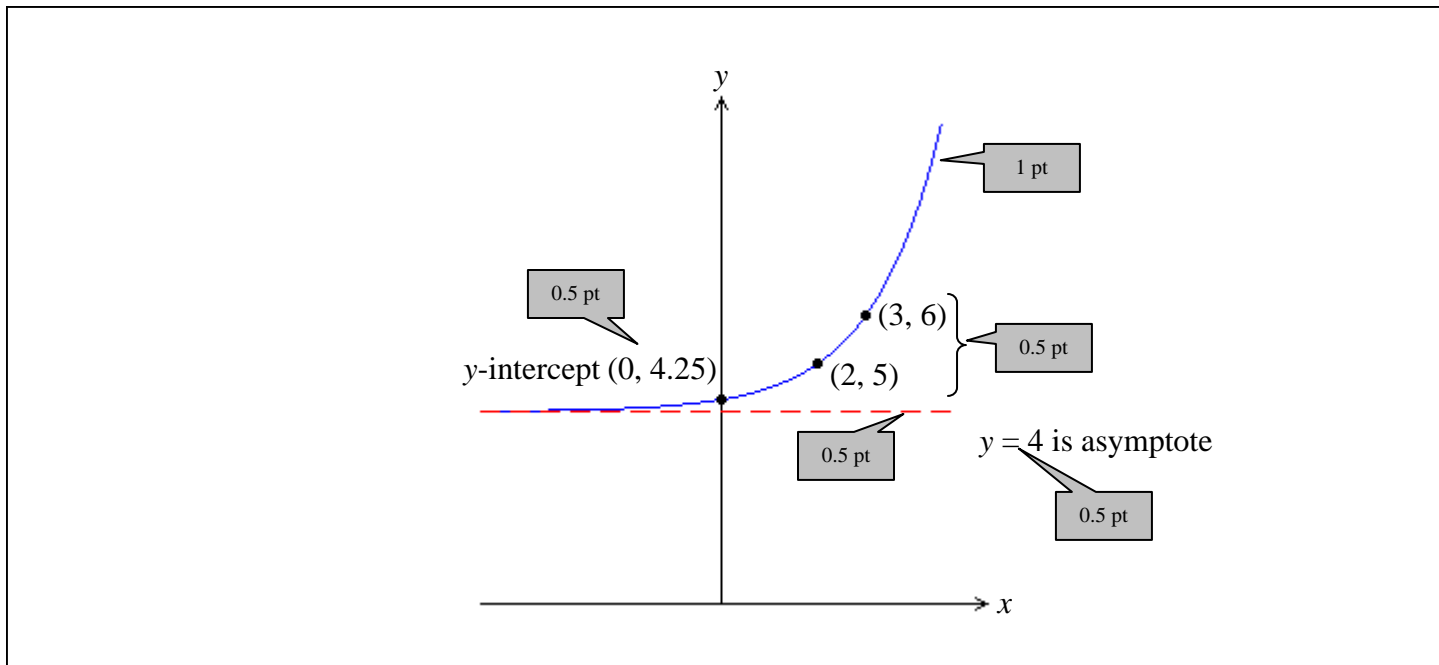
Question 33 (b)   (1 point)

The function is a growth curve because, for $y = ab^x + c$, $a > 0$ and $b > 1$.

Question 33 (c)   (1 point)

Domain: $x \in \mathbb{R}$ or $(-\infty, \infty)$ 0.5 pt

Range: $\{ y \mid y > 4, y \in \mathbb{R} \}$ or $(4, \infty)$ 0.5 pt

Question 34 (5 points)

Country A : $P_A = 40(1.06)^n$ — 1 pt

Country B : $P_B = 60(1.04)^n$ — 1 pt

$40(1.06)^n = 60(1.04)^n$ — 0.5 pt

$\log\left[40(1.06)^n\right] = \log\left[60(1.04)^n\right]$

$\log 40 + n\log 1.06 = \log 60 + n\log 1.04$ — 1 pt

$n\log 1.06 - n\log 1.04 = \log 60 - \log 40$

$n(\log 1.06 - \log 1.04) = \log 60 - \log 40$

$n = \dfrac{\log 60 - \log 40}{\log 1.06 - \log 1.04}$ — 0.5 pt

$n \doteq 21.29$ — 1 pt

The populations will be the same 21.29 years later.

$40(1.06)^n = 60(1.04)^n$ — 0.5 pt

$\dfrac{1.06^n}{1.04^n} = \dfrac{60}{40}$ — 0.5 pt

$\left(\dfrac{1.06}{1.04}\right)^n = 1.5$ — 0.5 pt

$n\log\left(\dfrac{1.06}{1.04}\right) = \log 1.5$

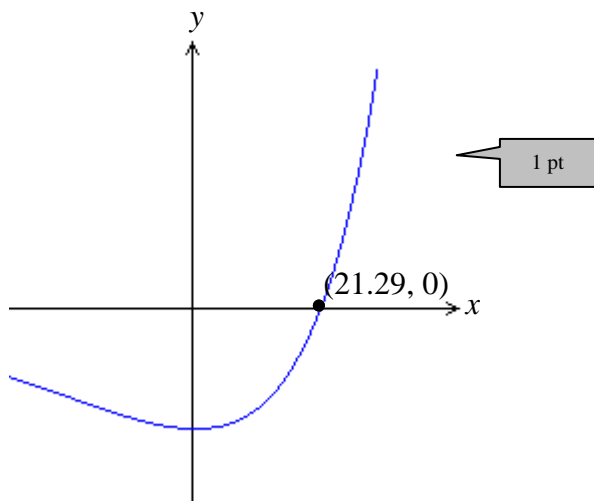$n = \dfrac{\log 1.5}{\log\left(\dfrac{1.06}{1.04}\right)}$ — 0.5 pt

$n \doteq 21.29$ — 1 pt

The populations will be the same 21.29 years later.

OR

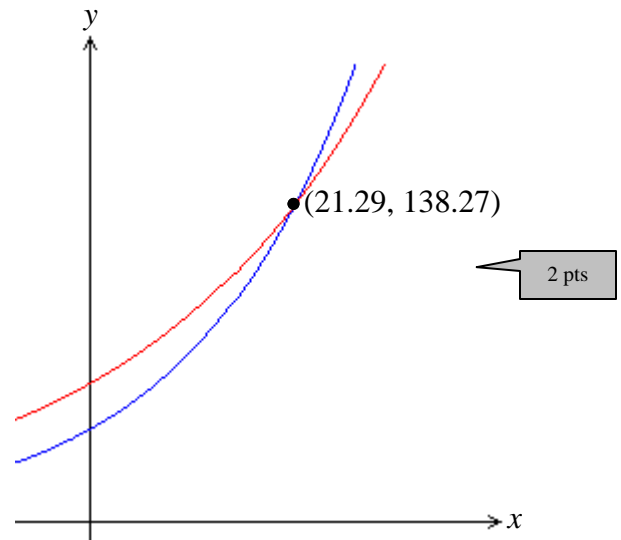$y = 40(1.06)^x - 60(1.04)^x$ — 1 pt

1 pt

(21.29, 0)

The populations will be the same 21.29 years later. — 1 pt

$y_1 = 40(1.06)^x$

$y_2 = 60(1.04)^x$

(21.29, 138.27)

2 pts

The populations will be the same 21.29 years later. — 1 pt

13

Question 35 (a)   (1 point)

(i) $\log_2 8 = 3$ ◁ 0.5 pt   (ii) $-\log_{\frac{1}{2}} 8 = 3$ ◁ 0.5 pt

Question 35 (b)   (1 point)

(i) $\log_3 9 = 2$ ◁ 0.5 pt   (ii) $-\log_{\frac{1}{3}} 9 = 2$ ◁ 0.5 pt

Question 35 (c)   (1 point)

$$-\log_{\frac{1}{b}} N = \log_b N$$ ◁ 1 pt

Question 36   (3 point)

These are examples of acceptable answers. Others are possible.

1 pt   1 pt   1 pt

✓ The value, *P*, of an investment of $1000 which earns 6% per year compounded semi-annually for *t* years.

1 pt   1 pt

✓ The number of bacteria, *P*, in a Petri dish increases by 3% every half day for *t* days. The initial number of bacteria is 1000.

1 pt

14

Question 37 (a)　(1 point)

Examples of acceptable answers (each worth 1 pt):

- ✓ ...they are equidistant from the centre.
- ✓ ...they subtend arcs of equal length.
- ✓ ...they subtend equal angles.

Question 37 (b)　(1 point)

Examples of acceptable answers (each worth 1 pt):

- ✓ ...it passes through the centre.
- ✓ ...it bisects the chord at right angles.

Question 38　(3 points)

0.5 pt　0.5 pt　0.5 pt

$$\left(\frac{x-5}{9}\right)^2 + \left(\frac{y-4}{5}\right)^2 = 1$$　0.5 pt

0.5 pt　0.5 pt

Question 39　(3 points)

Midpoint of diameter : $\left(\dfrac{-1+13}{2},\ \dfrac{3+5}{2}\right)$　　　$D_{radius} = \sqrt{(6+1)^2 + (4-3)^2}$

$(6,\ 4)$　1 pt　　　$= \sqrt{50}$　1 pt

Distance from centre to (11, 9):　$D = \sqrt{(6-11)^2 + (4-9)^2}$

$= \sqrt{50}$　1 pt

(11, 9) is on the circle, because the circle is the set of all points $\sqrt{50}$ from (6, 4)　1 pt

Question 40   (5 points)

Statement(s)	Reason(s)	
$\overline{PO} \cong \overline{SO}$	Given	
$\angle POQ \cong \angle SOR$	Vertically opposite angles	
$\overline{OQ} \cong \overline{OR}$	Given	
$\triangle PQO \cong \triangle SRO$	SAS	1 pt
$\overline{PQ} \cong \overline{SR}$	CPCTC	1 pt
$\overline{PR} \cong \overline{SQ}$	Segment addition	1 pt
$\overline{QR} \cong \overline{QR}$	Common side	1 pt
$\triangle PQR \cong \triangle SRQ$	SSS	1 pt

OR

Statement(s)	Reason(s)	
$\overline{PO} \cong \overline{SO}$	Given	
$\overline{OQ} \cong \overline{OR}$	Given	
$\overline{PR} \cong \overline{SQ}$	Segment addition	1 pt
$\triangle OQR$ is an isosceles triangle	$\overline{OQ} \cong \overline{OR}$	1 pt
$\angle OQR \cong \angle ORQ$	Base angles of an isosceles triangle	1 pt
$\overline{QR} \cong \overline{QR}$	Common side	1 pt
$\triangle PQR \cong \triangle SRQ$	SAS	1 pt

Question 41 (3 points)

1 pt

$$\frac{_5C_2}{_{15}C_2} = \frac{10}{105} \text{ or } \frac{2}{21}$$

1 pt

1 pt

OR

1 pt

$$\frac{5 \times 4}{15 \times 14} = \frac{20}{210} \text{ or } \frac{2}{21}$$

1 pt

1 pt

Question 42 (a) (2 points)

1 pt

0.5 pt

$$\frac{_4C_4 \times_{11}C_3}{_{15}C_7} = \frac{165}{6435} \text{ or } \frac{1}{39} \text{ or } 0.03$$

0.5 pt

Question 42 (b) (2 points)

1 pt

$$\frac{_{11}C_5}{_{13}C_5} = \frac{462}{1287} \text{ or } \frac{14}{39} \text{ or } 0.36$$

0.5 pt

0.5 pt

Question 43 (2 points)

Examples follow, other responses may be accpetable:

- ✓ What is the probability of being 6'3" or left-handed?

- ✓ What is the probability of becoming a teacher or a mother?